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The Leonid meteor storms of 1833 and 1966

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ABSTRACT

The greatest Leonid meteor storms since the late eighteenth century are generally regarded as being those of 1833 and 1966. They were evidently due to dense meteoroid concentrations within the Leonid stream. At those times, the orbit of Comet 55P/Tempel–Tuttle was significantly nearer that of the Earth than at most perihelion returns, but still some tens of Earth radii away. Significantly reducing this miss distance can be critical for producing a storm. Evaluation of differential gravitational perturbations, comparing meteoroids with the comet, shows that, in 1833 and 1966 respectively, the Earth passed through meteoroid trails generated at the 1800 and 1899 returns.

Key words: comets: individual: 55P/Tempel–Tuttle – meteors, meteoroids.

1 PREDICTING LEONID STORMS

The most spectacular meteor storms of the past 200 years were probably the Leonids of 1833 and 1966 (Kresák 1993a,b; Rao 1998). The parent comet of the Leonids, 55P/Tempel–Tuttle (Table 1), presently has orbital period ≈ 33.2 yr, and storms occur in years around the perihelion passages of the comet.

However, the occurrence of storms is not the same at every return to perihelion. First, the period of the comet not being a near-integer number of years immediately shows that the Earth–comet configuration is not repeated at successive returns. Moreover, the distance between the orbits of comet and Earth changes slightly over an orbital period. This distance, together with the time lag between comet and Earth passing their near-intersection point (meteoroids affected by radiation pressure tending to fall behind the comet), has been used as quite a good predictor of enhanced meteor activity (Yeomans 1981; Rao 1998).

The storms, the greatest of which can produce 10^4 times the meteor rate of Leonids in normal years, and even 10^2 times that of some years that are themselves classified as storm years, show that there are narrow, dense concentrations within the Leonid stream. They presumably comprise material on orbits close to that of the comet, relative to the scale of the stream as a whole. If many meteoroids had been ejected on to orbits further away, intervening regions of orbital element parameter space would also be filled, rather than narrow, high-density regions existing in space. Thus this material is recently released, not having had time to disperse throughout the stream. These compact trails of meteoroids and dust were discovered in the orbits of other periodic comets by *IRAS* (Sykes & Walker 1992), and their potential to produce meteor storms when the Earth intersection geometry is right has been emphasized by Kresák (1993a,b).

One trail is created during each return of the comet, surviving until it is dispersed into the stream. As these are distinct entities from the comet itself (although they are related), considering their separate orbital evolution could give greater accuracy for predicting the most intense storms than does consideration of the comet's orbit. For a few revolutions, enough trail material may stay coherent enough that systematic differences from the orbit of the comet can be precisely calculated, these differences dominating the randomness that arises because the exact initial orbits of individual particles at ejection are unknown.

The Leonid stream as a whole is not studied here [see Brown & Jones (1996) for a fuller model of the stream]. That is, while a model meteor flux is often generated statistically in dynamical meteoroid stream studies because the flux results from a wider range of stream orbits than can be explicitly considered individually, or because the time-scale is long enough for the chaotic dynamics which affects all planet-approaching orbits to occur, the idea here is to consider just a few individual trails and to follow particles on short enough time-scales that orbits are predictable.

2 DIFFERENTIAL PERTURBATIONS

The heliocentric distance r_D of the descending node is

$$r_D = \frac{a(1 - e^2)}{1 - e \cos \omega}, \qquad (1)$$

while the distance of the Earth at the longitude where it passes through the Leonid stream is $r_E \approx 0.9886$ au (200 yr ago, $r_E \approx 0.9889$ au, precession having marginally altered the value of the relevant longitude). As the solar–antisolar components of the velocity vectors of Leonid and Earth are both small near this point, the difference between r_D and r_E approximately gives the orbit–orbit approach distance. For reference, one Earth diameter is about 0.0001.

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Table 1. The orbit of 55P/Tempel–Tuttle calculated by Nakano (1998): epoch 1998 March 8.0 TT (Terrestrial Dynamical Time). The transverse non-gravitational parameter A_2 is more reliably determined than the radial component A_1 (cf. table I of Yeomans, Yau & Weissman 1996).

Perihelion passage time	$T = 1998$ Feb. 28.0982 TT
Perihelion distance	$q = 0.976577$
Semi-major axis	$a = 10.337486$
Eccentricity	$e = 0.905531$
Inclination (J2000.0)	$i = 162.4860$
Longitude of ascending node	$\Omega = 235.2583$
Argument of perihelion	$\omega = 172.4988$
Non-gravitational parameters (au d^{-2})	$A_1 = -0.80 \times 10^{-8}$ $A_2 = +0.0090 \times 10^{-8}$

From (1),

$$\frac{\partial r_\mathrm{D}}{\partial \omega} = -\frac{a(1-e^2)e\sin\omega}{(1-e\cos\omega)^2} \approx -0.001\,\mathrm{au\,deg}^{-1},$$

where appropriate a, e, ω have been used to give the numerical value. Similarly, writing (1) as

$$r_\mathrm{D} = \frac{q(2-q/a)}{1-(1-q/a)\cos\omega}$$

and using $a \approx 10$, $q \approx 1$ and $\cos\omega \approx -0.988$,

$$r_\mathrm{D} \approx q \Rightarrow \frac{\partial r_\mathrm{D}}{\partial q} \approx 1$$

and

$$\frac{\partial r_\mathrm{D}}{\partial a} \approx 3 \times 10^{-5}$$

can soon be derived. Hence

$$\Delta r_\mathrm{D} \approx \Delta q + 3 \times 10^{-5}\Delta a - 0.001\Delta\omega \qquad (2)$$

(cf. Pecina & Šimek 1997).

Although meteoroids released on to similar orbits to the comet will undergo similar orbital evolution for a while, the question arises as to whether even rather small differences in elements could crucially affect how closely a meteoroid can approach Earth. This can be investigated by trial integrations that need only cover quite short time-scales.

Only meteoroids with the right mean anomaly M to impact Earth at whatever date of interest need be considered (cf. Wu & Williams 1996). Specifying at which perihelion passage a meteoroid is ejected, and a given year not too many revolutions later in which it produces a meteor, tightly constrains the value a_0 of the semimajor axis at ejection, since the time elapsed between ejection and meteor impact fixes the orbital period. The period is not taken to be constant, gravitational perturbations being known to change the semimajor axis of the comet by amounts of order 0.1 au between successive returns, but, because the perturbations over a short enough time-scale are a rather smooth function of a_0, just a few iterations (integrations) allow a_0 to be determined. The key point is that, although the Δa term is negligible in (2) for relevant Δq, Δa and $\Delta\omega$ (Section 4), variations in a are the primary cause of the differential planetary perturbations that lead to differences in all the elements. Furthermore, those perturbations, and so the heliocentric distance (through Δq and $\Delta\omega$) and longitude of the descending node after a few revolutions, are precisely calculable (by means of integrations) functions of the initial elements (primarily a_0), if there are no close approaches to planets.

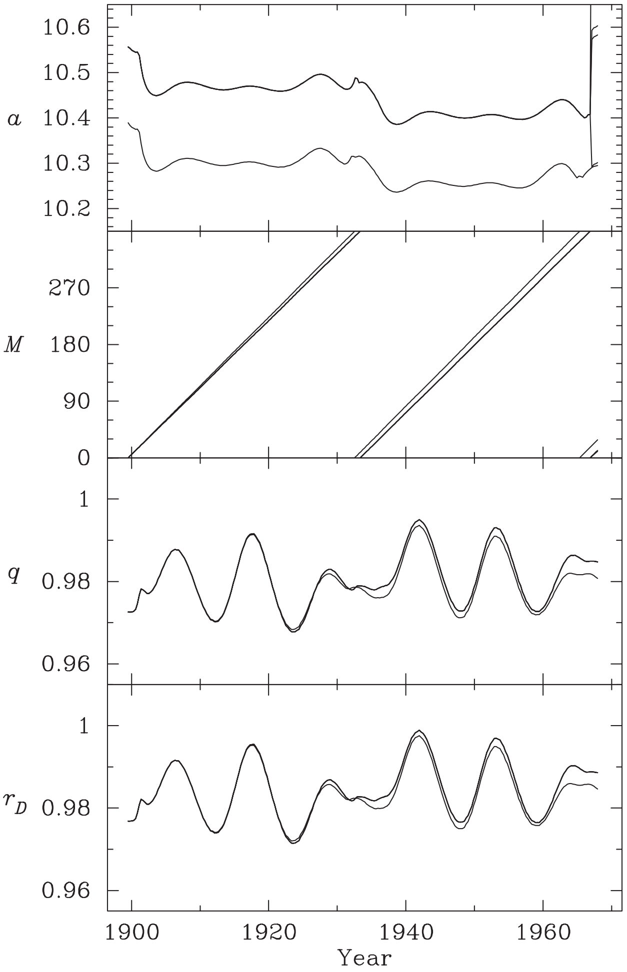

Figure 1. Orbital element evolution, for just over two revolutions, beginning at perihelion in 1899, of 55P/Tempel–Tuttle and four particles with initial q, i, Ω and ω matching the comet but different a (comet 10.39, particles 10.5564 to 10.5567).

Everhart's (1985) 15th-order Radau integrator has been used, in a program by Chambers & Migliorini (1997), with initial planetary elements in each integration taken from the Jet Propulsion Laboratory's DE403. The conclusions below can be found using an accuracy parameter of 10^{-8} in Radau and including Mercury in the Sun, allowing the computer programs to be run quickly, but as computer time was available, the results in Table 2 were generated with 10^{-9} and by explicitly integrating Mercury (altogether taking ~ 6 times longer).

Fig. 1 shows an example of differential perturbations between comet and meteoroids. The fact that the individual meteoroids cannot be resolved until the Earth approach in 1966 causes their semimajor axes to separate (initial orbits were specifically chosen because they undergo this approach) confirms that, over just a couple of revolutions, the effect of planetary perturbations, in particular on r_D, is quite deterministic (chaotic dynamics affects evolution after the close approach). As the range in mean anomaly, M, in 1966 brackets the M that would give Earth impact, checking r_D tells us whether impact actually occurs. This example shows the significant differences that can result from the perturbations, r_D moving away from the cometary value by some tens of Earth diameters.

Table 2. Nodal crossing points of recently (⩽ 3 revolutions previously) ejected particles that go through the ecliptic at the same time as the Earth passes through the Leonid stream in the given year, and of 55P/Tempel–Tuttle when it passes through the ecliptic (which happens between the second and third of each set of six times). It is the values relating to the particles that are physically meaningful with regard to assessing whether meteor storms can occur. Listed are the difference Δa_0 in semimajor axis from the comet at the time of ejection 1, 2 or 3 revolutions before; the difference between the heliocentric distances of Earth and particles at the longitude of the particles' descending node; and the longitude of the ascending node (which equals the longitude of the Sun, since the crossing point in question is the descending node). The two values of Δa_0 are for $\beta = 0$ (purely gravitational motion) and $\beta = 0.001$ (the ratio of the radiation pressure force to the gravity of the Sun). Changing β had little effect on $r_\mathrm{E} - r_\mathrm{D}$ and Ω. The value of Ω is relative to the standard equinox J2000.0 for ease of comparison with data generally in the literature (e.g. table 1 of Kresák 1993a), but the nodal crossing is through the ecliptic of the date as the deviation from the J2000 ecliptic can lead to values differing by e.g. $\sim 0\overset{\circ}{.}035$ in 1900 and $\sim 0\overset{\circ}{.}07$ in 1800. The omitted data were unreliable because the relevant particles were significantly perturbed by Earth during an intervening perihelion passage. Especially low values of $|r_\mathrm{E} - r_\mathrm{D}|$, the miss distance from Earth, are expected to be associated with storms. Smaller $|\Delta a_0|$ may also be advantageous, in the sense of needing smaller ejection speeds from the comet.

	1 revolution before				2 revolutions before				3 revolutions before				Comet	
Year	Δa_0 (au)		$r_\mathrm{E} - r_\mathrm{D}$ (au)	Ω (deg)	Δa_0 (au)		$r_\mathrm{E} - r_\mathrm{D}$ (au)	Ω (deg)	Δa_0 (au)		$r_\mathrm{E} - r_\mathrm{D}$ (au)	Ω (deg)	$r_\mathrm{E} - r_\mathrm{D}$ (au)	Ω (deg)
1798	−0.28	−0.48	0.0043	233.04	−0.15	−0.36	0.0057	233.02	−0.09	−0.31	0.0017	232.15		
1799	−0.07	−0.28	0.0032	233.04	−0.04	−0.25	0.0036	233.03	−0.02	−0.24	0.0018	232.84		
1800	0.14	−0.08	0.0028	233.03	0.07	−0.15	0.0020	233.06	0.02	−0.19	0.0061	233.33	0.0030	233.04
1801	0.35	0.12	0.0029	233.02	0.19	−0.04	0.0007	233.17	0.06	−0.16	0.0105	233.58		
1802	0.56	0.32	0.0030	233.04	0.31	0.08	−0.0011	233.50	0.09	−0.13	0.0134	233.95		
1803	0.76	0.52	0.0022	233.18	0.42	0.19	−0.0009	234.44	0.12	−0.10	0.0209	234.84		
1831	−0.25	−0.44	0.0034	233.16	−0.13	−0.34	0.0051	233.17	−0.10	−0.31	0.0068	233.16		
1832	−0.04	−0.24	0.0014	233.17	−0.02	−0.23	0.0017	233.18	−0.02	−0.23	0.0019	233.18		
1833	0.17	−0.04	−0.0003	233.18	0.09	−0.12	−0.0016	233.18	0.07	−0.15	−0.0028	233.21	0.0012	233.18
1834	0.38	0.16	−0.0017	233.18	0.20	−0.01	−0.0043	233.17	0.15	−0.07	−0.0070	233.27		
1835	0.59	0.36	−0.0026	233.18	0.31	0.09	−0.0065	233.17	0.24	0.01	−0.0107	233.42		
1836	0.79	0.55	−0.0033	233.18	0.42	0.20	−0.0083	233.18	0.33	0.10	−0.0142	233.81		
1864	−0.25	−0.44	0.0124	233.97	−0.13	−0.33	0.0138	233.94	−0.10	−0.31	0.0156	233.95		
1865	−0.04	−0.24	0.0072	233.32	−0.02	−0.23	0.0074	233.32	–	–	–	–		
1866	0.17	−0.04	0.0036	233.30	0.09	−0.12	0.0026	233.31	0.07	−0.15	0.0012	233.31	0.0065	233.30
1867	0.37	0.15	−0.0002	233.42	0.20	−0.01	−0.0026	233.43	0.15	−0.07	−0.0057	233.42		
1868	0.58	0.35	0.0011	234.06	0.31	0.10	−0.0045	234.03	0.24	0.02	−0.0096	234.01		
1869	0.78	0.54	0.0102	233.43	0.43	0.21	0.0055	233.49	0.32	0.10	−0.0006	233.54		
1897	−0.35	−0.54	−0.0020	234.24	−0.18	−0.37	0.0008	234.93	−0.12	−0.32	0.0013	235.27		
1898	−0.14	−0.34	0.0155	234.84	−0.07	−0.28	0.0167	234.97	−0.05	−0.26	0.0176	235.05		
1899	0.07	−0.14	0.0138	235.03	0.04	−0.17	0.0132	234.98	0.03	−0.18	0.0126	234.98	0.0117	234.63
1900	0.28	0.06	0.0199	234.07	0.15	−0.06	0.0182	234.02	0.11	−0.10	0.0167	234.05		
1901	0.48	0.25	0.0146	233.85	0.25	0.04	0.0124	233.82	0.19	−0.02	0.0096	233.85		
1902	0.68	0.44	0.0114	233.85	0.36	0.14	0.0086	233.85	0.28	0.06	0.0035	234.03		
1930	−0.36	−0.55	0.0075	235.10	−0.17	−0.38	0.0071	235.25	−0.12	−0.32	0.0019	235.39		
1931	−0.14	−0.35	0.0065	235.09	−0.08	−0.29	0.0105	234.90	−0.06	−0.26	0.0125	235.09		
1932	0.07	−0.15	0.0060	235.09	0.03	−0.18	0.0060	235.36	0.02	−0.18	0.0060	235.44	0.0061	235.08
1933	0.28	0.06	0.0054	235.15	0.11	−0.11	0.0119	236.01	0.07	−0.14	0.0135	235.99		
1934	0.49	0.25	0.0040	235.50	0.16	−0.06	0.0173	236.25	0.10	−0.11	0.0182	235.95		
1935	0.69	0.45	0.0183	235.97	0.23	0.01	0.0342	236.21	0.16	−0.05	0.0327	235.66		
1963	−0.31	−0.51	0.0059	235.09	−0.17	−0.37	0.0077	235.11	−0.13	−0.33	0.0136	235.10		
1964	−0.10	−0.30	0.0038	235.12	−0.05	−0.26	0.0043	235.12	−0.04	−0.25	0.0063	234.95		
1965	0.11	−0.10	0.0023	235.13	0.06	−0.16	0.0017	235.14	0.04	−0.17	0.0015	235.45	0.0031	235.13
1966	0.32	0.10	0.0015	235.13	0.17	−0.05	−0.0002	235.16	0.09	−0.13	0.0034	235.94		
1967	0.53	0.30	0.0012	235.13	–	–	–	–	0.13	−0.09	0.0063	236.21		
1968	0.73	0.49	0.0010	235.15	0.39	0.17	−0.0037	235.44	–	–	–	–		
1996	−0.28	−0.47	0.0099	235.30	−0.15	−0.35	0.0126	235.27	−0.11	−0.32	0.0149	235.27		
1997	−0.06	−0.27	0.0085	235.26	−0.04	−0.24	0.0091	235.26	−0.03	−0.24	0.0095	235.26		
1998	0.14	−0.07	0.0068	235.26	0.08	−0.13	0.0055	235.27	–	–	–	–	0.0081	235.26
1999	0.35	0.13	0.0047	235.28	0.19	−0.02	0.0019	235.27	0.14	−0.08	−0.0007	235.29		
2000	0.55	0.33	0.0031	235.29	0.30	0.08	−0.0012	235.27	0.22	0.00	−0.0050	235.32		
2001	0.76	0.52	0.0021	235.29	0.41	0.19	−0.0034	235.25	0.30	0.08	−0.0087	235.39		

At aphelion, 55P/Tempel–Tuttle reaches approximately the orbit of Uranus, but Jupiter and Saturn are its main perturbers (Yeomans et al. 1996). The differential perturbations in Fig. 1 are, in the main, not induced during the outer part of the orbit. The orbit of Saturn is crossed at around 35° and 325° in *M*; significantly beyond this, the evolutions of comet and meteoroids are almost indistinguishable in the first revolution, and separated but parallel in the second. (The slow oscillations while the objects are away from the inner Solar system, about two cycles being apparent in each revolution, are because elements are heliocentric, and Jupiter causes the barycentric position of the Sun to oscillate with period ~ 12 yr.) As the comet and meteoroids return towards perihelion for the first time, they are already several degrees apart in *M*, which can correspond to well over an astronomical unit, and the second time the difference is greater still. This causes perturbations to differ. Conversely, if two particles remain close together in *M*, then even if (Section 3) they are subject to moderately different radiation pressures, their orbital precessions will be similar.

3 EARTH ENCOUNTER CIRCUMSTANCES

Further examples show r_D moving away from the cometary value by greater or lesser amounts than in Fig. 1. For various years near the returns of the comet, particles were found with the right M to produce a meteor (a necessary but not sufficient condition to do so), to see if their nodal crossing points were also near meteor-producing values (Table 2). At first, the comet was integrated, including the non-gravitational parameters, to give elements at each return. The main integrations then involved starting at one perihelion time (in turn, one, two or three revolutions earlier) and integrating four particles, covering a small range in a_0, displaced from the a-value of the comet with other elements fixed, forward an approximately whole number of revolutions until the relevant date in November. The relative behaviour of four particles confirms that, over these fairly short time-scales, the elements are generally well-behaved functions of the initial elements, and thus that the a_0 found really is the value that gives the right M rather than being random, and therefore that significance can be attached to the values of r_D and Ω determined. A few iterations usually converged on the desired a_0, which is listed in Table 2 as Δa_0, i.e. relative to the cometary value at ejection.

Naturally, all particles found in each final iteration approach Earth, so that their evolution afterwards would become less predictable, but as it was predictable until that point, the determination of their nodal points was reliable. Only a few cases were badly behaved strictly before the encounter; having multiple particles was then useful to highlight something being wrong. The partial absence of data for 1865, 1967, 1968 and 1998 is because meteoroids that would have had an appropriate a_0 came quite close to Earth in respectively 1832, 1933, 1934 and 1965 (the 1934 case was the least disruptive, perhaps because of greater orbital distance from Earth). For example, the ‘missing’ value for 1865 is $\Delta a_0 \approx -0.02$, which appears (as the ‘2 revolutions’ entry) against 1832. Thus these encounters perturb the orbital elements erratically, whereas usually, for some revolutions, the elements evolve as smooth functions of the elements at ejection.

The calculations were repeated with a non-zero value of β, the ratio of the forces of radiation pressure and solar gravity, $\beta = 0.001$ being reasonable for meteoroids that produce visual Leonids (Williams 1997). This had virtually no effect on r_D and Ω at the accuracy given in Table 2; a_0 was changed as shown, the two values being consistent with what is expected (Williams 1997), and giving a reasonable idea of what Δa_0 would be for other β.

The Table 2 calculations had ejection at perihelion, meaning that q and thus r_D are not affected by non-zero β. In fact, a particle with $\beta = 0.001$ on an unperturbed elliptical orbit will have q (defining this, as with a_0, as being calculated from instantaneous position and velocity with $GM_\odot$ taking its usual value, not modified by β) 0.001 au greater at perihelion than away from perihelion [cf. equation (2) of Yeomans 1981]. More than half of the difference occurs within $r < 2$ and so $\beta = 0.001$ meteoroids ejected at $r \sim 2$ could have r_D increased by $\gtrsim 0.0005$ au. Repeating some integrations (a column from Table 2), this time beginning 100 d before perihelion, and checking the r_D values, confirms this. Depending on the meteoroid production rate as a function of r, the average increase in r_D relative to Table 2 will be of the order of one to a few times 0.0001. This is much less than the r_D range arising from differential gravitational perturbations (Table 2), which are therefore primarily what bring orbits to Earth intersection. Insofar as radiation pressure is important in affecting r_D for visual meteor size particles, it is more indirectly, through changing the orbital period with the consequent effect on differential gravitational perturbations, than directly in pushing particles out towards Earth-intersecting orbits. Quite often radiation pressure, in increasing the period, happens to lead to perturbations that decrease r_D.

Whether values in Table 2 signify major storms depends on $r_\mathrm{E} - r_\mathrm{D}$ being as small as possible, or certainly within the trail width. Also the values of β must apply to the real meteoroids, and Δa_0, for appropriate β, must be within an acceptable range. Δa_0 near zero will always be a good fit since most ejection scenarios will tend to produce a range of a_0 centred on the parent. The permitted (positive or negative) Δa_0 depends on ejection processes (Section 4). Here, radiation pressure can move the value of Δa_0 nearer to zero and will tend to make better meteor displays lag the comet, as is well known.

The small values of $r_\mathrm{E} - r_\mathrm{D}$ for the 1800 and 1899 trails respectively producing meteors in 1833 and 1966 are quite striking. In both cases, 1966 especially, the miss distance is smaller than that based on the comet, the values of Ω (cf. Kresák 1993a; Mason 1995) being correct either way. Although in a typical year it is not the case that most meteors have originated at a single perihelion passage, there is evidence that such was the case in those two great storm years.

The only other similarly small $r_\mathrm{E} - r_\mathrm{D}$ is for 1867, a storm year but lesser than 1833 and 1966. The 1833 trail could be largely responsible for the 1867 storm, meteoroids either having somewhat larger β, or being somewhat away from the central values of a_0 produced on ejection (the latter would impose a lower bound on ejection speeds needed). The values of Ω in the reasonable storm years 1866–68 match observations quite well, the 1868 value especially differing from the comet. Depending on what spread in elements within a trail is reasonable, Table 2 might be able to explain why, despite the comet's similar r_D, Leonid activity was greater in 1866–68 than around the 1933 return.

As trails gradually lengthen in M, the spatial density will tend to become progressively diluted with each revolution, reducing the potential meteor flux. Moreover, increasingly many sections of the trail will be disrupted. Nevertheless, nodal crossings were checked, for a few years of possible interest, for larger numbers of revolutions (up to six) between ejection and appearance of meteors. It was found that the 1600 trail gave an excellent nodal distance match ($\lesssim 0.0001$) in 1799 (i.e. six revolutions), the values for four and five revolutions being similar to those for three shown in Table 2. It is hard to be sure of the relative importance of the trails in producing the impressive 1799 storm, but a clearly better fit of Ω (well constrained by eyewitness reports: Rao 1998) than the cometary value was notable. For reference, the calculated Ω is 232.80. Regarding the similarly good 1832 storm, four, five and six revolutions all give $r_\mathrm{E} - r_\mathrm{D} \approx 0.0010$.

No especially close nodal distances are found for any trail and 1998 meteors, but a distance of 0.0008 is found for the 1866 trail and 2000 meteors (four revolutions). For 1999 meteors, the 1899 trail listed in Table 2 is much closer than for the three earlier trails. Despite the similar preferred values of β to the 1833 and 1966 cases, both the greater miss distance and the greater number of revolutions make a storm at the level of those two unlikely. However, there may be some kind of storm, the estimated distance from the centre of the trail being less than 10 Earth diameters. It may also be desirable to calculate as accurately as possible the three-dimensional position of the trail as a function of time over a day or so around predicted passage near (through) it, so as to assess whether there is a risk to satellites (cf. Beech, Brown & Jones 1995).

One year of possible interest that is not in Table 2 is 1969, when a sharp outburst, notable for being rich in small particles

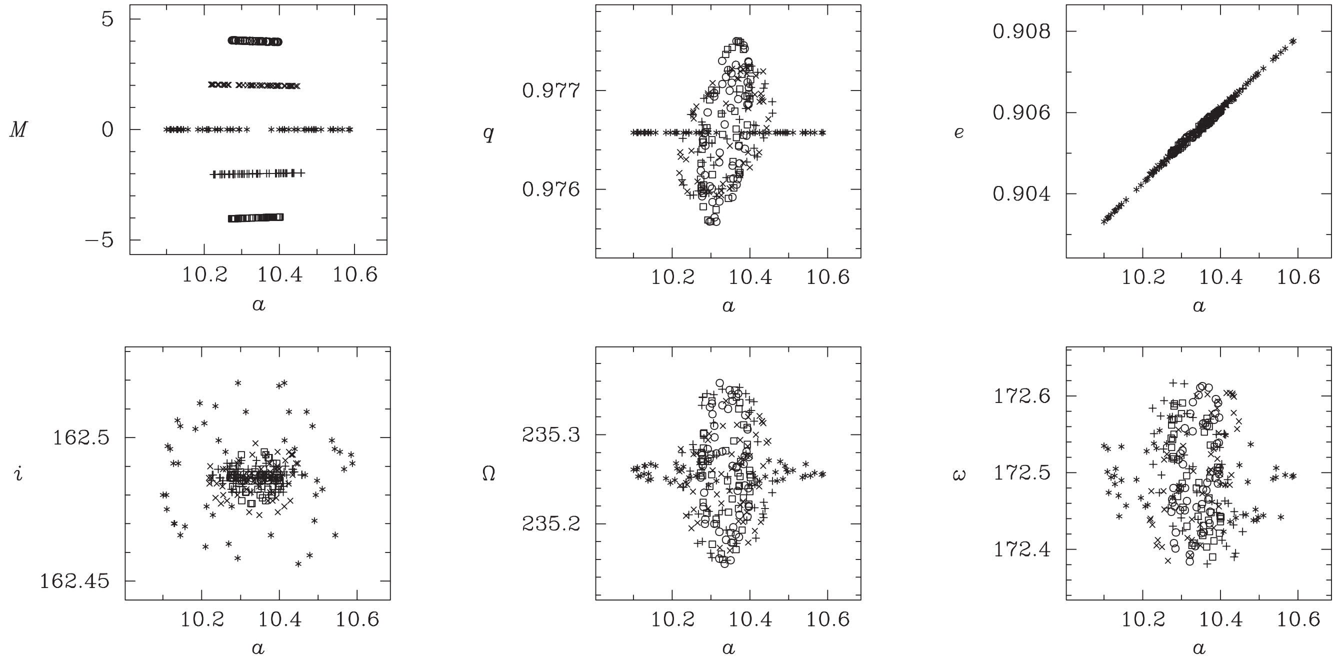

Figure 2. Orbital elements of 250 test particles randomly (isotropically) ejected from 55P/Tempel–Tuttle at its 1998 return, 50 at each of $M = -4°$, $-2°$, $0°$, $2°$ and $4°$ (plotted as squares, pluses, asterisks, crosses and circles). Ejection velocities of 25, 15 and $10\,\mathrm{m\,s^{-1}}$ at $r = 1$, 1.5 and 2.2 au were adopted.

(McIntosh 1973), was observed. It turns out that the 1932 trail gives both a perfect nodal distance match ≈ 0.0000 and $\Omega = 235.27$ precisely in accord with the timing of the peak (Millman 1970). The required Δa_0 is 0.9 au if $\beta = 0$, with $\beta = 0.004$ giving $\Delta a_0 = 0$. Such a value of β corresponds to small particles. This fit impressively demonstrates the applicability of the trail model.

4 DISPERSION

In addition to radiation pressure causing position to vary with particle size, the range in orbital elements induced on ejection affects the extent of each trail. The Poynting–Robertson effect is negligible at this early evolutionary stage in a stream (Kresák 1993b).

Some repeat integrations (a column from Table 2 at a time) were done but in each case altering one of the initial elements q, i, Ω and ω (respectively by 0.002, 0.02, 0.2 and 0.2). To a very good approximation, differences in these elements were conserved over the time-scales in question. The range in elements at ejection time is therefore of interest as it carries through to a range in r_D (and Ω) at meteor observation time via equation (2).

A detailed investigation, e.g. using analytical formulae for changes in elements (Pecina & Šimek 1997) in addition to the Monte Carlo type procedure used for Fig. 2, or considering a range of physical models (cf. Brown & Jones 1998), is not attempted at this stage, but Fig. 2 gives an approximate idea of spreads in elements that may be expected. For simplicity, ejection is isotropic (rather than having the sunward hemisphere only, or jets opposite to the known sense of the transverse non-gravitational force). Consideration of cometary ejection processes (Whipple 1951; Jones 1995) suggests that the velocities in Fig. 2 are reasonable, given a nuclear radius ≈ 1.8 km (Hainaut et al. 1998). There are correlations among elements that will not be explored here, but the overall dispersions should tell us something useful.

The range in Ω of a trail gives the length of the Earth's path through it, which is a factor of ~ 3 longer than the vertical cross-section of the stream, since the Earth passes through the Leonid stream at an angle of 17°, this angle being largely in a vertical plane (cf. first paragraph of Section 2). The Ω range in Fig. 2 corresponds to a few hours, consistent perhaps with some past storms. The width across the Earth's path depends via equation (2) on q and ω. In Fig. 2, the former converts to $\gtrsim 0.001$ au in cross-section and the latter rather less. In the absence of further modelling, ~ 0.001 au may be tentatively adopted as the cross-section. Increases in density towards the centre would be unsurprising. The distance is reasonably in accord with the attempts to interpret the $r_\mathrm{E} - r_\mathrm{D}$ distances from Table 2 in Section 3. For visual meteor size particles, the random dispersion owing to ejection seems clearly less than the range in r_D arising from perturbations, emphasizing the value of accurately calculating the latter. It can be noted that the range in a and both cross-sections will all tend to increase and decrease together as the maximum ejection velocity is varied in the model.

Studying past meteor activity profiles (Jenniskens 1995) would be useful. Observations from 1966 may yield information about the 1899 trail, perhaps of interest before 1999 November.

5 DUST TRAILS AND METEOR STORMS

The purpose of this paper has been to do the calculations that identify as easily as possible the {ejection time, meteor year} pairs likely to produce the greatest storms. Kondrat'eva, Murav'eva & Reznikov (1997) were the first, so far as I am aware, to make a significant number of such identifications relating to the Leonids, but, as that paper was quite short, it has been worth presenting here an independent derivation and discussion not given there.

More detailed modelling of the lengthening and then disruption of each trail (cf. Kresák 1993a,b) could be done. Before a trail is disrupted, perturbations at a distance from Jupiter and Saturn are probably responsible for its gradual evolution away from the orbit of the comet. Although small, this difference from the orbit of 55P/Tempel–Tuttle has a critical effect on the occurrence of meteor storms. The actual disruption of cometary trails is generally due to the giant planets (Kresák 1993a,b), but the Earth also affects the Leonids, since its orbit passes through the stream. During each perihelion passage, a total of as much as 10 per cent (comprising separate trail sections a year apart) of each trail can be seriously affected by the Earth. Williams (1997) has shown how, in the present few centuries, a near-commensurability makes trails near the comet unusually safe from disruption by Uranus. Whatever the cause of disruption, the various trails eventually merge into the background Leonid stream.

Given records of meteor storms over the past millennium (Hasegawa 1993; Mason 1995), this study could be extended to see if they can be related to particular trails. The orbit of 55P/Tempel–Tuttle may be known accurately enough for this purpose. This could help to confirm a general lifetime for coherent, dense trails of the order of a few revolutions.

Numerical integrations, then, can be used to find the nodal crossing points of those parts of trails that cross the ecliptic when the Earth is nearby. These locations can be determined at the accuracy given in Table 2, and need not necessarily match the point where the comet crosses the ecliptic. The occurrence of a storm is dependent on the distance from the Earth being small enough. Based both on relating calculated miss distances to past storms, which has successfully explained the observed storms with the highest zenithal hourly rate (Section 3), and on estimates of orbital element dispersion (Section 4), the allowed miss distance for a storm appears to be a few times 0.0001 au. A procedure therefore exists to determine reliably when Leonid storms occur.

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REFERENCES

Beech M., Brown P., Jones J., 1995, QJRAS, 36, 127
Brown P., Jones J., 1996, in Gustafson B. Å. S., Hanner M. S., eds, ASP Conf. Ser. Vol. 104, Proc. IAU Colloq. 150, Physics, Chemistry and Dynamics of Interplanetary Dust. Astron. Soc. Pac., San Francisco, p. 113
Brown P., Jones J., 1998, Icarus, 133, 36
Chambers J. E., Migliorini F., 1997, BAAS, 29, 1024
Everhart E., 1985, in Carusi A., Valsecchi G. B., eds, Proc. IAU Colloq. 83, Dynamics of Comets: Their Origin and Evolution. Reidel, Dordrecht, p. 185
Hainaut O. R., Meech K. J., Boehnhardt H., West R. M., 1998, A&A, 333, 746
Hasegawa I., 1993, in Štohl J., Williams I. P., eds, Meteoroids and their Parent Bodies. Slovak Academy of Sciences, Bratislava, p. 209
Jenniskens P., 1995, A&A, 295, 206
Jones J., 1995, MNRAS, 275, 773
Kondrat'eva E. D., Murav'eva I. N., Reznikov E. D., 1997, Sol. Syst. Res., 31, 489
Kresák Ľ, 1993a, A&A, 279, 646
Kresák Ľ., 1993b, in Štohl J., Williams I. P., eds, Meteoroids and their Parent Bodies. Slovak Academy of Sciences, Bratislava, p. 147
McIntosh B. A., 1973, in Hemenway C. L., Millman P. M., Cook A. F. eds, NASA SP–319, Evolutionary and Physical Properties of Meteoroids. NASA, Washington DC, p. 193
Mason J. W., 1995, J. Brit. Astron. Assoc., 105, 219
Millman P. M., 1970, J. R. Astron. Soc. Can., 64, 55
Nakano S., 1998, Minor Planet Circ., 31070
Pecina P., Šimek M., 1997, A&A, 317, 594
Rao J., 1998, WGN, 26, 192
Sykes M. V., Walker R. G., 1992, Icarus, 95, 180
Whipple F. L., 1951, ApJ, 113, 464
Williams I. P., 1997, MNRAS, 292, L37
Wu Z., Williams I. P., 1996, MNRAS, 280, 1210
Yeomans D. K., 1981, Icarus, 47, 492
Yeomans D. K., Yau K. K., Weissman P. R., 1996, Icarus, 124, 407

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